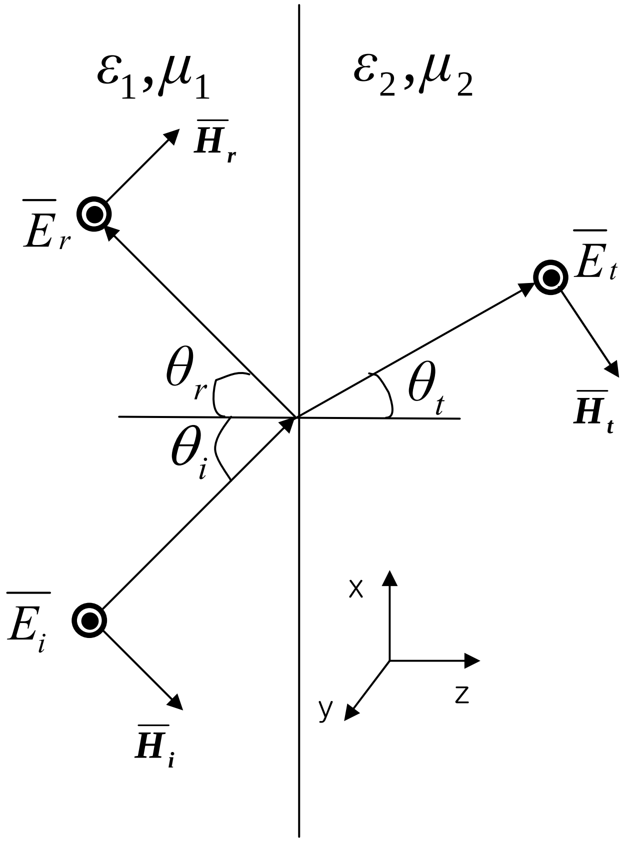
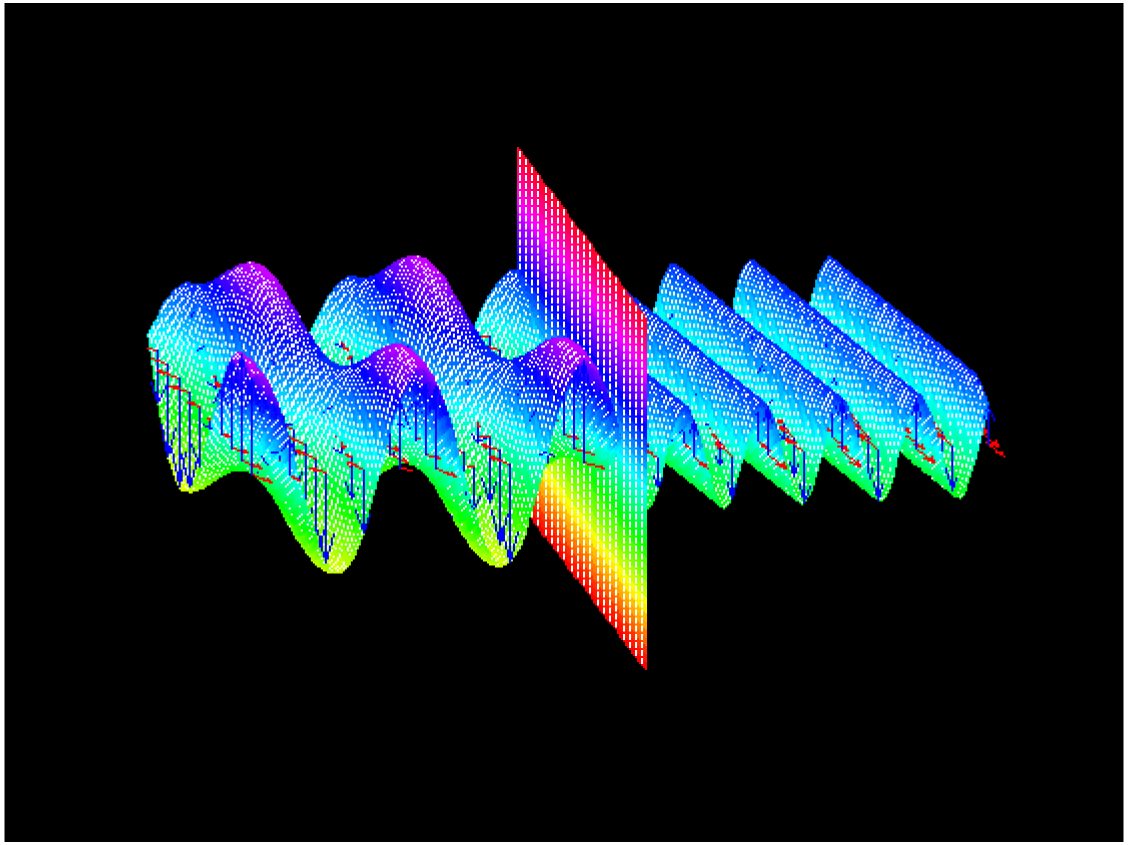
Lect. 3: Reflection and Transmission

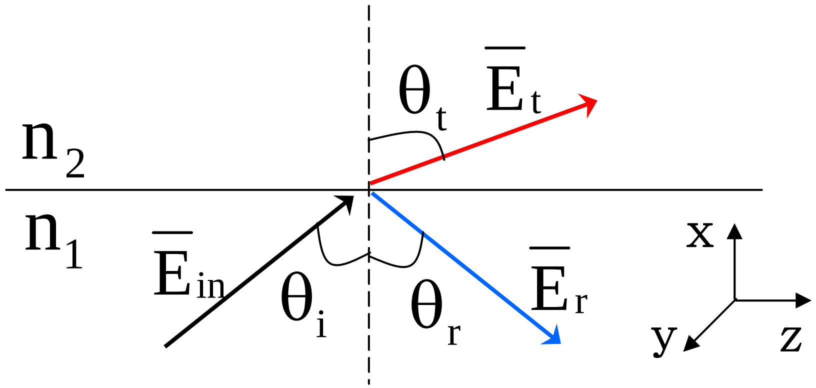




$4\varepsilon_1 = \varepsilon_2,\ \mu_1 = \mu_2$ and $\theta_i = 20^\circ$

Lect. 3: Reflection and Transmission

(Assuming perpendicular polarization)



$$\overline{E}_{in} = \overline{y}E_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{E}_r = \overline{y}E_r e^{jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{E}_t = \overline{y}E_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

$$k_x = n_1 k_0 \cos\theta_i, k_z = n_1 k_0 \sin\theta_i$$

$$k_{rx} = n_1 k_0 \cos\theta_r, k_{rz} = n_1 k_0 \sin\theta_r$$

$$k_{tx} = n_2 k_0 \cos\theta_t, k_{tz} = n_2 k_0 \sin\theta_t$$

$$\theta_r, \theta_t, E_r, E_t = ???$$

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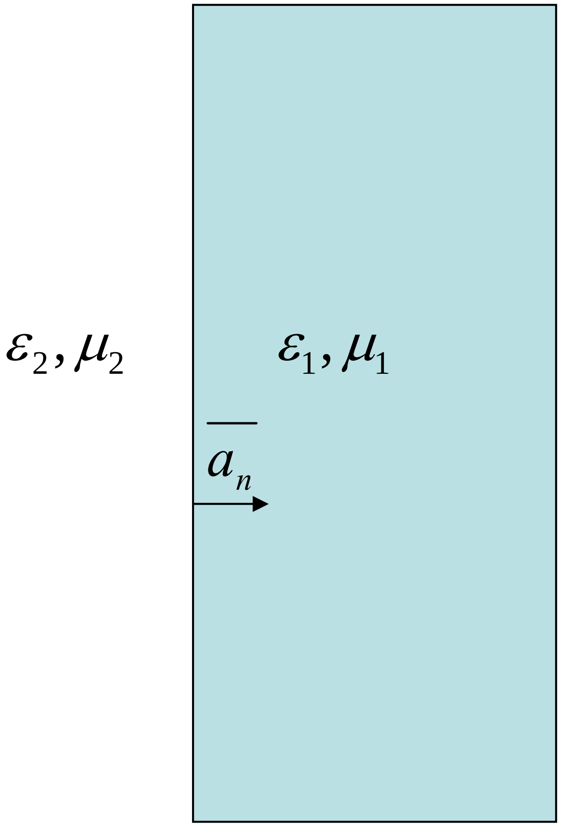
✓ Review of Boundary Conditions: Constraints on E,H fields at a boundary. Each Maxwell's Eq. provides one constraint on E or H.



$$\nabla \bullet \overline{D} = \rho,$$

$$D_{1,n} - D_{2,n} = \rho_s \Leftrightarrow \varepsilon_1 E_{1,n} - \varepsilon_2 E_{2,n} = \rho_s$$

$$\nabla \times \overline{E} = -\frac{d\overline{B}}{dt},$$

$$\boxed{E_{1,t} - E_{2,t} = 0}$$

$$\nabla \bullet \overline{B} = 0,$$

$$B_{1,n} - B_{2,n} = 0 \Leftrightarrow \mu_1 H_{1,n} - \mu_2 H_{2,n} = 0$$

$$\nabla \times \overline{H} = \overline{J} + \frac{d\overline{D}}{dt},$$

$$\overline{a_n} \times (\overline{H}_1 - \overline{H}_2) = \overline{J}_s \Leftrightarrow \boxed{H_{1,t} - H_{2,t} = J_s}$$

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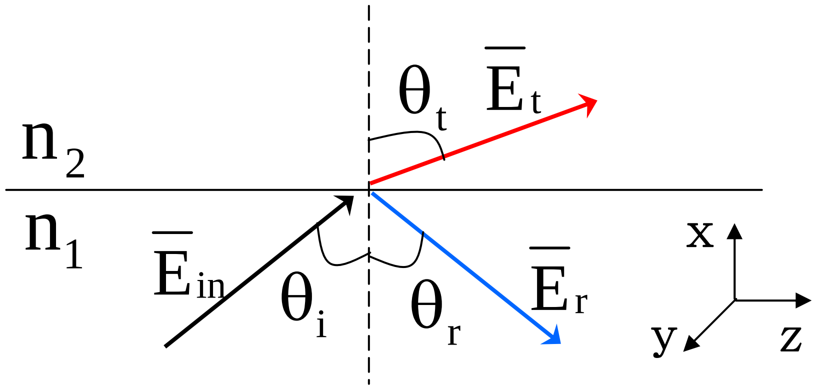

$$\overline{E}_{in} = \overline{y}E_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{E}_r = \overline{y}E_r e^{jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{E}_t = \overline{y}E_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

Applying BC on E at $x = 0$,

$$E_i e^{-jk_z \cdot z} + E_r e^{-jk_{rz} \cdot z} = E_t e^{-jk_{tz} \cdot z}$$

=> $\boxed{k_z = k_{rz} = k_{tz}}$ and $\boxed{E_i + E_r = E_t}$

$$\therefore\ n_1 \sin\theta_i = n_1 \sin\theta_r = n_2 \sin\theta_t$$

=> $\theta_i = \theta_r$ and $n_1 \sin\theta_i = n_2 \sin\theta_t$ (Snell's Law)

$$k_z = n_1 k_0 \sin\theta_i$$

$$k_{rz} = n_1 k_0 \sin\theta_r$$

$$k_{tz} = n_2 k_0 \sin\theta_t$$

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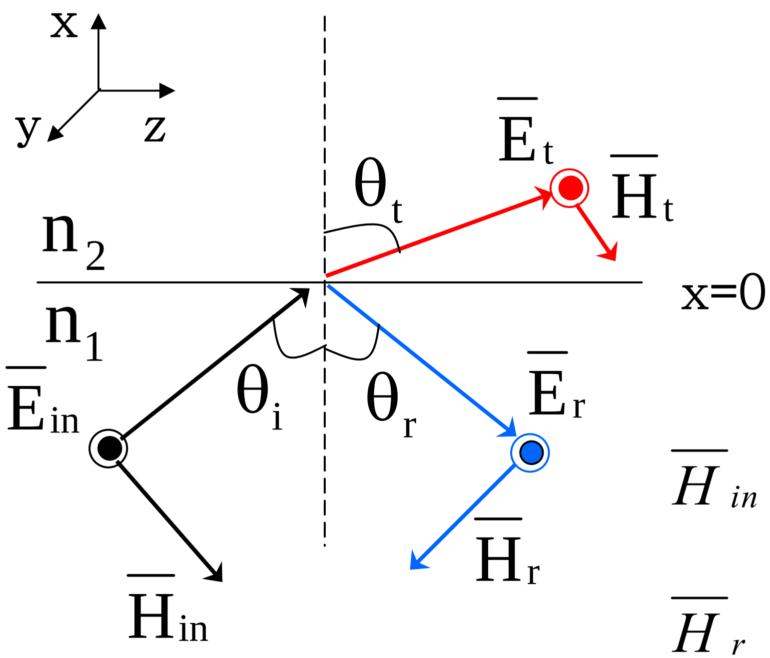

$$\overline{E}_{in} = \overline{y}E_i e^{-jk_x\cdot x}e^{-jk_z\cdot z}$$

$$\overline{E}_r = \overline{y}E_r e^{jk_{rx}\cdot x}e^{-jk_{rz}\cdot z}$$

$$\overline{E}_t = \overline{y}E_t e^{-jk_{tx}\cdot x}e^{-jk_{tz}\cdot z}$$

$$\overline{H}_{in} = \left(-\overline{x}\sin\theta_i + \overline{z}\cos\theta_i\right)\frac{E_i}{\eta_1} e^{-jk_x\cdot x}e^{-jk_z\cdot z}$$

$$\overline{H}_r = \left(-\overline{x}\sin\theta_i - \overline{z}\cos\theta_i\right)\frac{E_r}{\eta_1} e^{+jk_{rx}\cdot x}e^{-jk_{rz}\cdot z}$$

$$\overline{H}_t = \left(-\overline{x}\sin\theta_t + \overline{z}\cos\theta_t\right)\frac{E_t}{\eta_2} e^{-jk_{tx}\cdot x}e^{-jk_{tz}\cdot z}$$

Applying BC on H at x=0

$$\left(\cos\theta_i\right)\frac{E_i}{\eta_1} + \left(-\cos\theta_i\right)\frac{E_r}{\eta_1} = \left(\cos\theta_t\right)\frac{E_t}{\eta_2}$$

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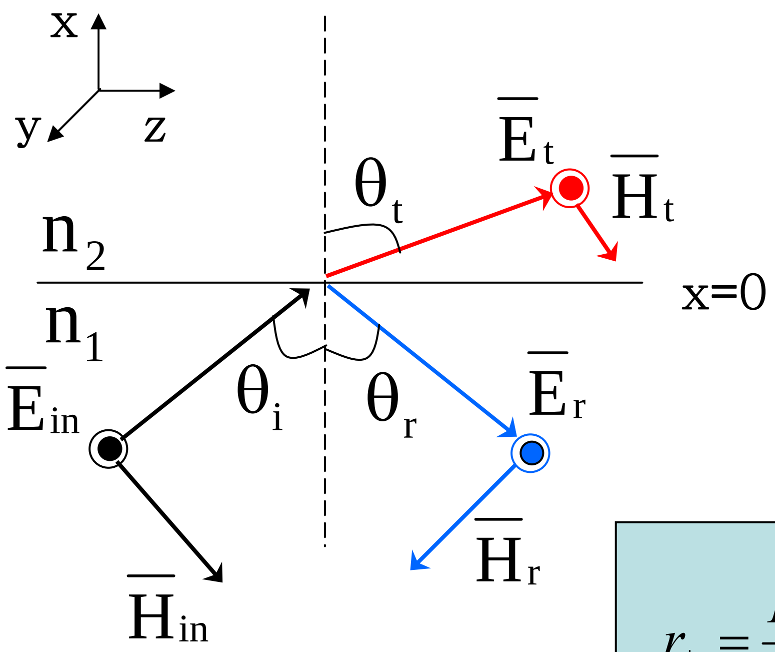

Two simultaneous equations:

$$E_i + E_r = E_t$$

$$(\cos\theta_i)\,{}^{E_i}\!/_{\eta_1} + (-\cos\theta_i)\,{}^{E_r}\!/_{\eta_1} = (\cos\theta_t)\,{}^{E_t}\!/_{\eta_2}$$

$$r_\perp = \frac{E_r}{E_i} = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

$$t_\perp = \frac{E_t}{E_i} = \frac{2\cos\theta_i}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}} \qquad (n = \frac{n_2}{n_1})$$

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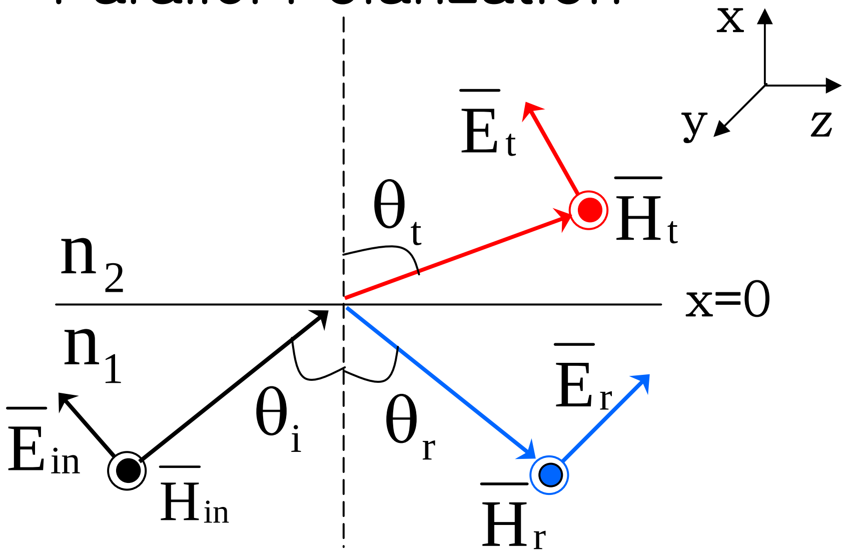
Parallel Polarization



$$\overline{H}_i = \overline{y} H_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

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$$\overline{H}_t = \overline{y} H_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

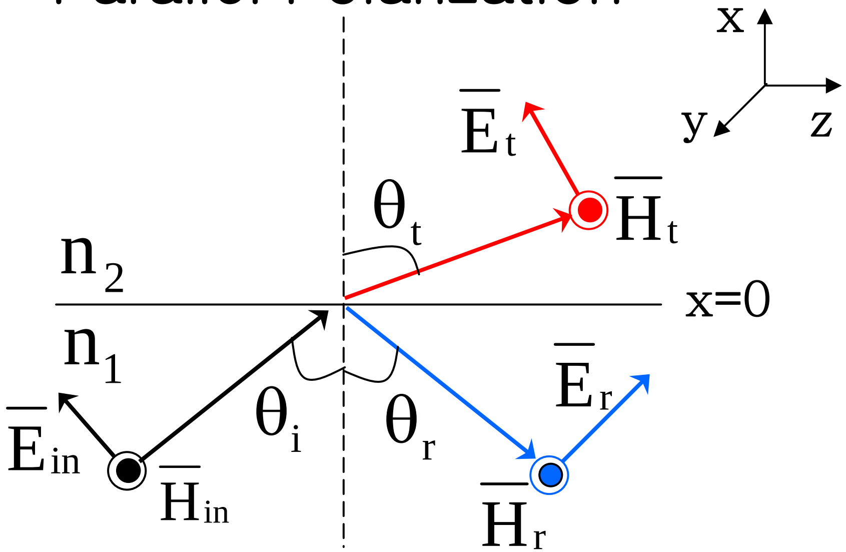
$$\overline{E}_i = \left(\overline{x} \sin\theta_i - \overline{z} \cos\theta_i\right) \cdot \eta_1 H_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\theta_r, \theta_t, H_r, H_t = ???$$

$$\overline{E}_r = \left(\overline{x} \sin\theta_r + \overline{z} \cos\theta_r\right) \cdot \eta_1 H_r e^{+jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{E}_t = \left(\overline{x} \sin\theta_t - \overline{z} \cos\theta_t\right) \cdot \eta_2 H_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

Parallel Polarization



$$\overline{H}_i = \overline{y} H_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$
$$\overline{H}_r = \overline{y} H_r e^{+jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$
$$\overline{H}_t = \overline{y} H_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

$$k_z = n_1 k_0 \sin\theta_i$$
$$k_{rz} = n_1 k_0 \sin\theta_r$$
$$k_{tz} = n_2 k_0 \sin\theta_t$$

Applying BC on H at $x = 0$,

$$H_i e^{-jk_z \cdot z} + H_r e^{-jk_{rz} \cdot z} = H_t e^{-jk_{tz} \cdot z}$$

=> $k_z = k_{rz} = k_{tz}$ and $H_i + H_r = H_t$

$\therefore\ n_1 \sin\theta_i = n_1 \sin\theta_r = n_2 \sin\theta_t$

=> $\theta_i = \theta_r$ and $n_1 \sin\theta_i = n_2 \sin\theta_t$ (Snell's Law)

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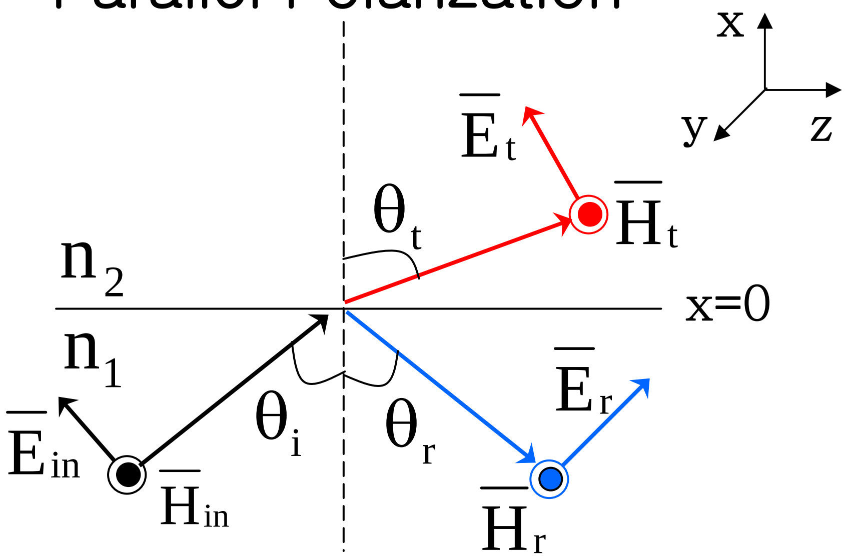
Parallel Polarization



$$\overline{E}_{in} = \left(\overline{x}\sin\theta_i - \overline{z}\cos\theta_i\right)\cdot \eta_1 H_i e^{-jk_x\cdot x} e^{-jk_z\cdot z}$$

$$\overline{E}_r = \left(\overline{x}\sin\theta_i + \overline{z}\cos\theta_i\right)\cdot \eta_1 H_r e^{+jk_{rx}\cdot x} e^{-jk_{rz}\cdot z}$$

$$\overline{E}_t = \left(\overline{x}\sin\theta_t - \overline{z}\cos\theta_t\right)\cdot \eta_2 H_t e^{-jk_{tx}\cdot x} e^{-jk_{tz}\cdot z}$$

Applying BC on E at $x = 0$,

$$-\cos\theta_i \cdot \eta_1 H_i + \cos\theta_i \cdot \eta_1 H_r = -\cos\theta_t \cdot \eta_2 H_t$$

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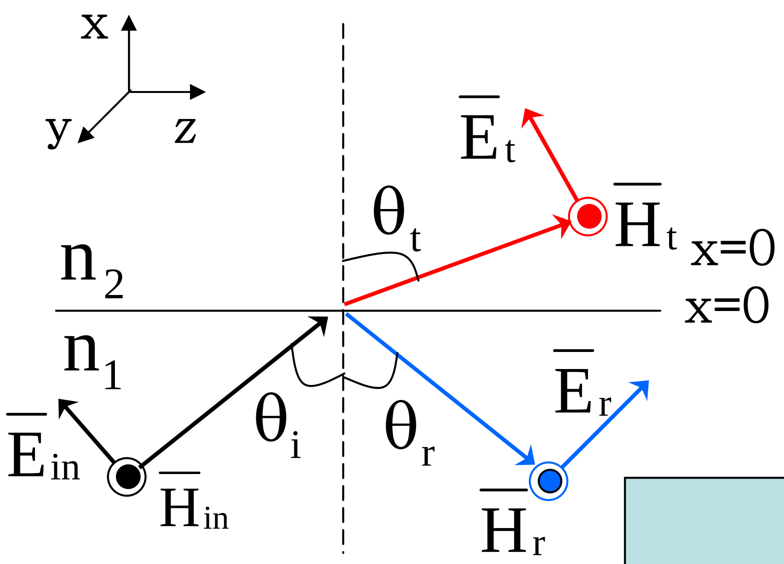

Two simultaneous equations:

$$H_i + H_r = H_t$$

$$-\cos\theta_i \cdot \eta_1 H_i + \cos\theta_i \cdot \eta_1 H_r = -\cos\theta_t \cdot \eta_2 H_t$$

$$r_{\parallel} = -\frac{H_r}{H_i} = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

$$t_{\parallel} = \frac{\eta_1 H_t}{\eta_2 H_i} = \frac{2n\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i} \qquad (n = \frac{n_2}{n_1})$$

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1st Quiz on next Tuesday

Exercise problems: Prob. 1, Prob. 11, Prob. 16